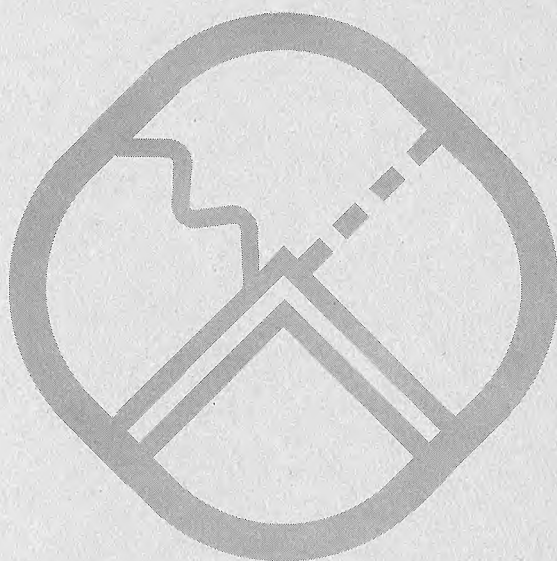


R.F. SYNCHRONIZATION DURING TRANSFER IN THE CASCADE SYNCHROTRON

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ABSTRACT

If a low energy synchrotron is to be used as an injector for a 300 Bev machine, the problem may arise of synchronizing the two R.F. systems at transfer time in order that a bunch of protons in the injector may be transferred to a bucket in the main machine without loss of particles. A possible synchronization scheme is proposed and investigated here.

ACKNOWLEDGMENTS

This work has resulted after discussions with Professors R. L. Walker and M. Sands.

The cascade synchrotron proposed by M. Sands¹⁾ would use as an injector for the main machine a 10 Gev AGS called the booster ring. We will consider here the problems encountered in synchronizing the R.F. system of the two machines during beam transfer. The parameters used here for the two machines are those given in Table II of Report CTSL-10.

The simplest mode of operation of the two machines would be to accelerate a bunch of particles at a repetition rate of 1 per second with the booster, transfer to the big machine and accelerate these protons to 300 Gev in the next second. However, as pointed out in Appendix A of Reference 1, this method of operation only fills about 3 per cent of the azimuthal phase space of the big machine. Courant, Snyder, and Walker have hence proposed operating the booster ring at a repetition rate of perhaps 10 cps and over a 3 second period injecting 30 bursts of particles into the main ring at a fixed field which fills its azimuthal phase space. The main ring then operates once every 3 seconds to carry these protons to full energy. The method of transfer to be described here is probably neither the only one possible, nor the best. It is presented mainly to illustrate that in this preliminary stage of study, no difficulties seem to be present that would dictate against the possibility of filling the azimuthal phase space in the main ring.

To be specific, we will assume the booster ring is working on its 20th harmonic. There are, therefore, 20 buckets filled with protons which must be transferred. We will operate the main ring at the same

1) M. Sands, Report CTSL-10, 1960.

initial frequency as the final frequency of the booster. Since the ratio of the radii of the two machines is 26, the initial frequency of the main ring will correspond to the 520th harmonic. There are, therefore, 520 buckets around the main ring which must be filled with 26 pulses consisting of 20 buckets each from the booster. The rotation period in the main ring is $26 \mu \text{ sec.}$ and in the booster $1 \mu \text{ sec.}$, and we will assume that the inflector can be turned on and off in $50 \text{ m } \mu \text{ sec.}$ and can be timed to somewhat greater accuracy. If this is the case, then one or two buckets at the beginning and end of each pulse from the booster will be lost. The transfer is basically accomplished by first locking the booster ring final frequency onto the main ring initial frequency. Second, adjusting the relative phase of the two r.f. systems so that after injection the 20 buckets of protons from the booster will fit into the centers of 20 buckets in the main ring, and third, timing the transfer so that the 520 buckets in the main ring will be filled in a systematic manner by 26 pulses from the booster.

We first investigate the problem of synchronizing the machines in frequency. Consider first that we have both machines with fixed field and their frequencies nearly the same so that they have a beat time of T_B . We assume T_B is long enough so that if the frequency of the booster is shifted into exact synchronism with f_0 , the frequency of the main ring, that the beam in the booster will still be within the useful aperture. We will use a simple version of beam control in the booster ring and begin by investigating the properties of this system alone.

For the system shown in Fig. 1, we have the following differential equations

$$\frac{d\epsilon}{dt} = -K f_B x \quad (1)$$

where x is the displacement from equilibrium orbit, f_B the rotation frequency of the booster, ϵ the change of energy for displacement x , and K is determined by the gain of the servo loop involving the difference electrodes, phase shifter, R.F. amplifier and accelerating electrode. It is equal to the energy per turn supplied to the beam for a unit displacement. In addition, we have the equations

$$\Delta p/p = v^2 \frac{x}{R_O} \quad \epsilon/E_O = \beta^2 \frac{\Delta p}{p} \quad (2)$$

Substituting, we find that

$$\frac{dx}{dt} = - \frac{K f_O R_O}{v^2 \beta^2 E_O} x \quad (3)$$

Thus the response of the beam in the booster ring to a step function signal inserted into point A is an exponential shift of the radius to a new equilibrium value with a time constant τ_B where

$$\tau_B = \frac{v^2 \beta^2 E_O}{K f_O R_O} \quad (4)$$

To see what the magnitude of this might be, let us suppose the gain of the loop is adjusted so that for a shift in radius of 1 cm, the R.F. system supplies 200 kev/turn. With the rest of the parameters as given in Table II of Report CTSL-10, we find

$$\tau_B = 300 \mu \text{ sec.} \quad (5)$$

Another way of stating this result is that with the gain used above, we may move the beam in radius by injecting signals at point A at a rate of about 3 cm/millisecond. As the maximum available R.F. in the booster ring is about 300 kev/turn, we must not require rates of change of the radius larger than that given above or the average phase angle of the bunch during the change will become too large and particles will be lost from the bucket.

Now consider the problem of synchronizing this R.F. system to a fixed frequency f_0 , represented by the main ring. In addition to matching the frequency, the phase must be adjusted so that when a full bucket from the booster has been ejected, traversed the distance between the two machines and has been inflected into the main ring, there will be a corresponding empty bucket matched to the incoming particles to perhaps $\pm 5^\circ$ of R.F. phase.

To accomplish this, the following scheme is visualized. Consider a phase detector whose two inputs are the R.F. voltages from the two separate systems. As the frequency of the booster approaches f_0 , there will be beats between the two R.F. systems and, hence, times when the two systems are in the correct phase but the wrong frequency. We utilize the phase detector output over its linear range as shown in Fig. 2. Now suppose $t = 0$ is defined by V_ϕ equal zero, and at this instance we connect this voltage to junction A, Fig. 1, so that the input to the phase shifter is the sum of the input of the difference electrodes and the phase detector output. We then have, in place of Eq. (1), the equation

$$\frac{d\phi}{dt} = -K f_0 x + \alpha f_0 (\phi_B - \phi_0) \quad (6)$$

where ϕ_B is the phase of the booster R.F. calculated from $t = 0$ by

$$\phi_B = \int_0^t \omega_B dt = \int_0^t \frac{v}{r_0} \left(1 - \frac{x}{r_0}\right) dt = \omega_0 \int_0^t \left(1 - \frac{x}{r_0}\right) dt \quad (7)$$

and where we have called r_0 in the booster that radius at which the frequency and energy are correct for injection to the main ring. ϕ_0 is the phase of the main ring R.F. and is given by

$$\phi_0 = \omega_0 t \quad (8)$$

α is a constant proportional to the sensitivity of the phase detector. Substituting (7) and (8) into (6), and making use of (2), we find

$$\frac{d^2x}{dt^2} + \frac{K f_0 R_0}{E_0 \beta^2 v^2} \frac{dx}{dt} + \frac{\alpha \omega_0^2}{E_0 \beta^2 v^2} x = 0 \quad (9)$$

We now choose α so that we have a critically damped system. The solutions then have a characteristic time constant given by

$$\tau = \tau_B/2 = \frac{1}{2} \frac{v^2 \beta^2 E_0}{K f_0 R_0} \quad (10)$$

and the solution is of the form

$$x = x_0 \left(1 - \frac{2t}{\tau_B}\right) e^{-2t/\tau_B} \quad (11)$$

Note that since the phases are correct at $t = 0$, that in order to shift frequency, x must necessarily have an over shoot. In fact, the area under the curve of x vs. t must be zero because the change in phase is zero and this as given by Eq. (7) and (8) is just the integral with respect to t of x . Also, as we have seen, $\tau_B/2$ is of the order of 150 μ sec., so 1 millisecond is ample time for any transient to decay.

Now we must consider under actual operating conditions whether at the peak of the booster magnetic field, enough time is available to accomplish this transfer, and also whether the stability of the booster is high enough so that successful transfer will be accomplished each cycle.

We will consider two different booster ring power supplies. One will consist of a condenser bank with the machine working at a 10 cps rate and the other will be a generator-flywheel combination working at the same repetition frequency. In the latter case, voltage inversion takes place by following a sine wave and hence, in both cases, the magnetic field is parabolic about the time at which the peak field is reached. Present power supplies have achieved a stability from pulse to pulse of less than 0.1 per cent and this could probably be improved if necessary. We will now consider what effect this fluctuation in B has on the transfer.

1. Betatron Oscillation.

If the energy at transfer is wrong, betatron oscillations will be induced in the orbits in the main ring. Since there is a static field in the main magnet, we will assume that this can be very accurately controlled and this field, together with the radius, defines an injection energy and frequency f_0 . Since the booster frequency will also be fixed to f_0 , we find that at inversion

$$\Delta p/p = \frac{\gamma^2}{\gamma^2 - v^2} \frac{\delta B}{B} \approx 1.3 \times 10^{-3} \quad (12)$$

This leads to a fluctuation of radius in the booster of

$$\Delta r_B = \frac{r_B}{v_B^2} \Delta p/p \approx 0.25 \text{ cm.}$$

This is perhaps a little large as this fluctuation, in position at transfer time, will give rise to position and angle errors in the beam injected into the main ring. However, it is still considerably smaller than the size of the beam itself. The betatron oscillations induced in the main ring by this fluctuation in momentum have an amplitude given by

$$\Delta r = \frac{r}{\gamma^2} \frac{\Delta p}{p} \approx 0.09 \text{ cm.}$$

which is again considered satisfactory. Poorly designed ejection - injection optics could, of course, increase this to a larger value.

2. Synchrotron Oscillations.

If we assume that the R.F. system in the main ring is operating at its full value of 12 Mev/turn, then the buckets at injection have a size of $\pm 180^\circ \times \pm 1 \text{ cm}$. This corresponds to a momentum acceptance of $\Delta p/p = .014$. During acceleration, the size is reduced to $\pm 0.43 \text{ cm} \times 135^\circ$. As this is very much larger than what is required to accommodate the momentum spread of the booster of $1 - 2 \times 10^{-4}$ ²⁾, it should be possible to reduce the R.F. voltage during this period to considerably below the maximum value. For instance, to accommodate twice the $\Delta p/p = 1.3 \times 10^{-3}$ given by Eq. (13), we need have only 400 kv/turn which gives a bucket size of $\pm 0.18 \text{ cm}$. This could not be used during acceleration, but would have to be increased to the full value. The accompanying adiabatic increase in radial synchrotron oscillation amplitude would amount to a factor of $(30)^{1/4} \approx 2.3$. However, this low voltage during the injection phase would reduce the power during this time by almost a factor of 1000.

2) R. L. Walker, Report CTSL-16, 1961.

3. Variations in Main Ring Field.

We have neglected variations in the field in the main ring. However, this may be a rather bad approximation to an actual magnet. However, for the first pulse injected, we do not need to have the main ring R.F. system operating. Since the momentum acceptance of the aperture is 3 per cent, which is very large compared to the fluctuations except from the booster, we could use beam control on the first pulse to establish the frequency f_0 and thus eliminate the need to precisely regulate the static field.

4. Time Available for Transfer.

The final point is to investigate the time available for transfer. For the generator-flywheel combination, operating at 60 cycles and inverting over approximately $1/2$ cycle, we have for $\Delta B/B \leq 10^{-3}$ a time available of 1.0 msec. For a condenser system operating at a 10 cps rate, we have $\Delta B/B \leq 10^{-3}$ for a period of 2.1 msec. These times are both considerably longer than the time necessary to lock the booster frequency to f_0 .

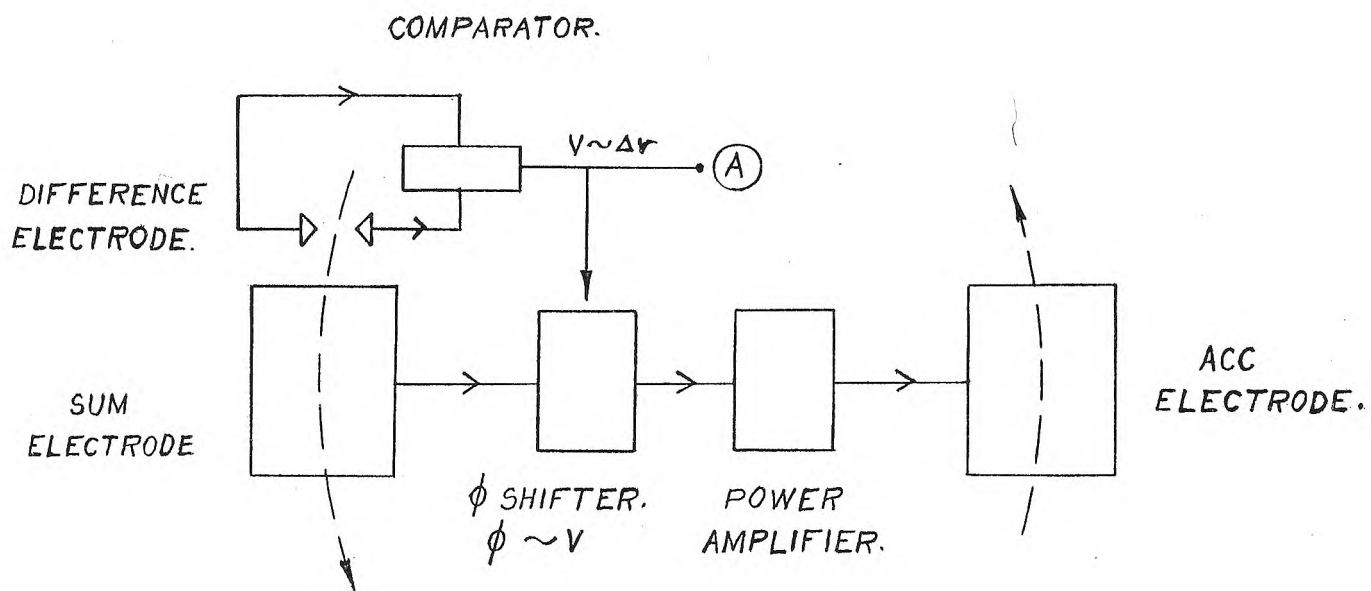


FIG. 1.

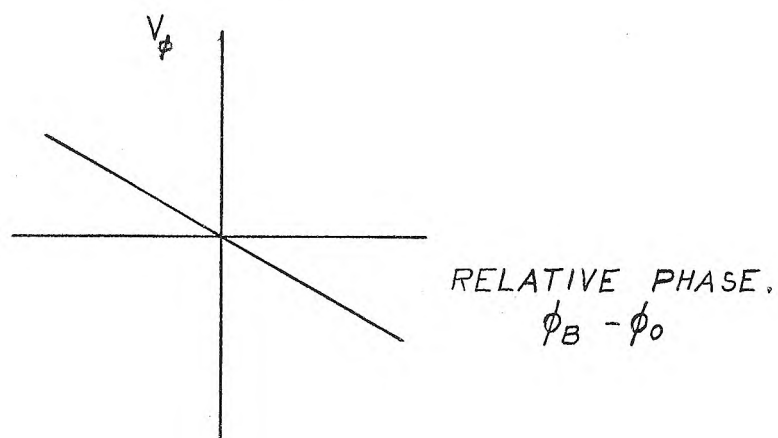


FIG. 2.

